

A comment on the new non-conventional gravitational mechanism proposed by Jaekel and Reynaud to accommodate the Pioneer anomaly

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Abstract

In this paper we put on the test the new mechanism of gravitational origin recently put forth by Jaekel and Reynaud in order to explain the Pioneer anomaly in the framework of their post-Einsteinian metric extension of general relativity. According to such a proposal, the secular part of the anomalous acceleration experienced by the twin spacecraft of about 1 nm s^{-2} could be caused by an extra-potential $\delta\Phi_P = c^2\chi r^2$, with $\chi = 4 \times 10^{-8} \text{ AU}^{-2}$, coming from the second sector of the considered model. When applied to the motion of the planets of the Solar System, it would induce anomalous secular perihelion advances which amount to tens-hundreds of arcseconds per century for the outer planets. As for other previously proposed non-conventional gravitational explanations of the Pioneer anomaly, the answer of the latest determinations of the anomalous perihelion rates by RAS IAA is neatly and unambiguously negative. The presence of another possible candidate to explain the Pioneer anomaly, i.e. the extra-potential $\delta\Phi_N$, linear in distance, from the first sector of the Jaekel and Reynaud model, is ruled out not only by the residuals of the optical data of the outer planets processed with the recent RAS IAA EPM2004 ephemerides but also by those produced with other, older dynamical theories like, e.g., the well known NASA JPL DE200.

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In the framework of their post-Einsteinian metric extension of general relativity, Jaekel and Reynaud (2006) recently put forth a new model which amends a previous one by the same authors (Jaekel and Reynaud 2005a; 2005b) and, among other things, yields a possible explanation of gravitational origin of the secular part of the anomalous acceleration of about 1 nm

s^{-2} experienced by the Pioneer 10/11 spacecraft in the range $20 \text{ AU} \lesssim r \lesssim 70 \text{ AU}$ (Anderson et al. 1998; 2002).

Basically, Jaekel and Reynaud (2006) start from a space-time line element

$$ds^2 = g_{00}(r)c^2 dt^2 + g_{rr}(r)(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2), \quad (1)$$

written in the standard Eddington isotropic coordinates, and write the metric coefficients as sums of standard relativistic expressions and small deviations

$$g_{\mu\nu} \equiv [g_{\mu\nu}]_{\text{st}} + \delta g_{\mu\nu}, \quad |\delta g_{\mu\nu}| \ll 1. \quad (2)$$

The two sectors $\delta g_{00}(r)$ and $\delta(g_{00}g_{rr})(r)$ yield two anomalous potentials $\delta\Phi_N$ and $\delta\Phi_P$ which affect the motion of test particles and light rays.

In this note we do not demand to discuss the model proposed by Jaekel and Reynaud (2006) in all of its generality, but only as far as possible explanations of the Pioneer anomaly are concerned.

1 On the first anomalous potential

In regard to the correction $\delta\Phi_N$ to the Newtonian potential coming from the first sector, Jaekel and Reynaud (2006) write: “Should the Pioneer anomaly be explained by an anomaly in the first sector, a linear dependence of the potential $\delta\Phi_N$ would be needed to reproduce the fact that the anomaly has a roughly constant value over a large range of heliocentric distances r_P

$$c^2 \partial_r \delta\Phi_N \simeq a_P, \quad 20 \text{ AU} \leq r_P \leq 70 \text{ AU}. \quad (3)$$

The simplest way to modelize the anomaly would thus correspond to a potential varying linearly with r and vanishing at Earth orbit [...]” In regard to the compatibility of such an extra-potential with the observed features of the planetary motions, especially in the regions in which the Pioneer anomaly manifested itself, according to our present-day knowledge of it, the predicted action of an anomalous constant and uniform, radial acceleration of about 1 nm s^{-2} on the orbits of the outer planets of the Solar System was investigated in the framework of the latest observations in a number of papers always getting neat and unambiguous negative answers.

Iorio and Giudice (2006) compared the time-dependent patterns of the directly observable quantities $\alpha \cos \delta$ and δ , where α and δ are the planetary right ascension and the declination, respectively, induced by a Pioneer-like extra-acceleration for Uranus (19.19 AU), Neptune (30.06 AU) and

Pluto (39.48 AU) to their observational residuals obtained by processing almost one century (1913-2003) of optical data with the RAS IAA EPM2004 ephemerides (Pitjeva 2005). Tangen (2006) did the same by using a different theoretical quantity. While a Pioneer-type force would affect $\alpha \cos \delta$ and δ with polynomial signatures of hundreds of arcseconds, the observed residuals are almost uniform strips well constrained within ± 5 arcseconds. It is interesting to note that the very same conclusion could already have been traced long time ago by using the residuals of some sets of modern optical observations (1984-1997) to the outer planets processed by Morrison and Evans (1998) with the NASA JPL DE405 ephemerides. Analysis of residuals obtained with even older ephemerides would have yielded the same results. For example, Standish (1993) used JPL DE200 ephemerides to process optical data of Uranus dating back to 1800: the obtained residuals of α and δ do not show any particular structure being well constrained within ± 5 arcseconds. Gomes and Ferraz-Mello (1987) used the VSOP82 ephemerides to process more than one century (1846-1982) of optical data of Neptune getting no anomalous signatures as large as predicted by the presence of a Pioneer-like anomalous force. In regard to Pluto, Gemmo and Barbieri (1994) and Rylkov et al. (1995) used the JPL DE200 and JPL DE202 ephemerides in producing residuals of α and δ : no Pioneer-type signatures can be detected in them.

Pitjeva (2006) recently determined the anomalous secular rates of perihelion ϖ for Jupiter (5.2 AU), Saturn (9.5 AU) and Uranus by contrasting, in a least-square sense¹, almost one century of mainly optical (apart from Jupiter) data with the full model of relevant Newtonian and Einsteinian dynamical effects of the not yet released EPM2006 ephemerides. After comparing them with the theoretical predictions for such precessions induced by a Pioneer-like acceleration, we got another clear negative answer, as pointed out in (Iorio 2006a; 2006b).

Even the use of the Voyager 2 radio-tracking data to Neptune ruled out the existence of a Pioneer-like acceleration which would affect the Neptune semi-major axis with a totally undetected short-period effect (Iorio 2006c; 2006b).

It is important to stress that such conclusions are purely phenomenological and model-independent: no speculations about the possible origin of such an extra-acceleration at all have been used.

In conclusion, we cannot agree with Jaekel and Reynaud (2006) when

¹Contrary to the right ascension and declination, the perihelia are not directly observable.

they write: “[...] it then remains to decide whether or not the ephemeris of the outer planets are accurate enough to forbid the presence of the linear dependence (3) in the range of distances explored by the Pioneer probes (Iorio and Giudice 2006; Tangen 2006). This point remains to be settled (Brownstein and Moffat 2006)”. It is just the case to note that, in fact, the gravitational mechanism put forth by Brownstein and Moffat (2006) by fitting all the presently available Pioneer 10/11 data to a Yukawa-type model² completely failed when applied to the perihelia of Jupiter, Saturn and Uranus (Iorio 2006d; 2006b).

2 On the second anomalous potential

In regard to the second sector, Jaekel and Reynaud (2006) write: “In any case, there is another possibility, namely that the Pioneer anomaly is induced by the second anomalous potential $\delta\Phi_P$ rather than the first one $\delta\Phi_N$. We now consider these terms which are still here even if there is no anomaly at all in the first sector ($\delta\Phi_N = 0$).” As a result of their investigation, Jaekel and Reynaud (2006) find that: “A roughly constant anomaly is produced when [...] $\delta\Phi_P(r)$ is quadratic in r , in the range of Pioneer distances.” Their choice is

$$\delta\Phi_P(r) = c^2\chi r^2, \quad \chi \simeq 4 \times 10^{-8} \text{ AU}^{-2}, \quad (4)$$

where c is the speed of light in vacuum. The resulting acceleration

$$A_P(r) = -2c^2\chi r, \quad (5)$$

in units of nm s^{-2} , is plotted in Figure 1. Without investigating how well such a model fits, in fact, all the currently available data of the Pioneer 10/11 spacecraft, here we are going to derive theoretical predictions for the secular perihelion advance induced by eq. (5). The standard methods of perturbative celestial mechanics yield

$$\frac{d\varpi}{dt} = -3c^2\chi \sqrt{\frac{a^3(1-e^2)}{GM}}, \quad (6)$$

where a and e are the semi-major axis and the eccentricity, respectively, of the planet’s orbit, G is the Newtonian gravitational constant and M is the mass of the Sun. Note that eq. (6) is an exact result. The comparison among the anomalous advances for Jupiter, Saturn and Uranus predicted with eq. (6) and the determined perihelia rates is in Table 1.

²Instead, Jaekel and Reynaud (2006) write that “Brownstein and Moffat have explored

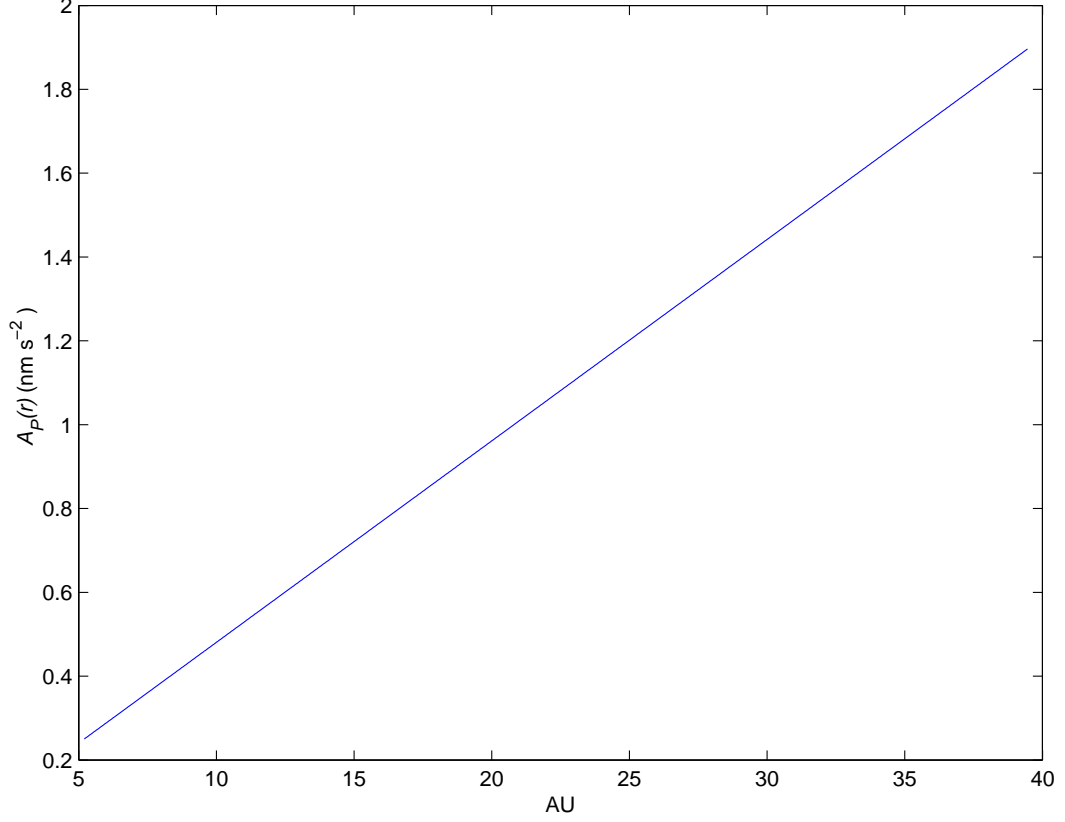


Figure 1: Anomalous acceleration, in nm s^{-2} , induced by $\delta\Phi_P = c^2\chi r^2$, with $\chi = 4 \times 10^{-8} \text{ AU}^{-2}$, according to Jaekel and Reynaud (2006).

Table 1: First row: determined extra-precessions of the perihelia of Jupiter, Saturn and Uranus, in arcseconds per century (Pitjeva 2006). The quoted uncertainties are the formal, statistical errors re-scaled by a factor 10 in order to get realistic estimates. Second row: predicted anomalous extra-precessions of the perihelia for Jupiter, Saturn and Uranus, in arcseconds per century, according to eq. (6).

	Jupiter	Saturn	Uranus
$\dot{\omega}_{\text{meas}}$	0.0062 ± 0.036	-0.92 ± 2.9	0.57 ± 13.0
$\dot{\omega}_{\text{pred}}$	-18.679	-46.3	-132.3

As can be noted, even by re-scaling by a factor 10 the formal errors released by Pitjeva (2006), the discrepancy among the predicted and the determined values amounts to 519, 15 and 10 sigma for Jupiter, Saturn and Uranus, respectively.

3 Conclusions

In this note we investigated the new proposal by Jaekel and Reynaud (2006) to accommodate the Pioneer anomaly in the framework of their post-Einsteinian metric extension of general relativity. First, we reviewed the wealth of observational evidence pointing against the presence in planetary data of any anomalous signature as large as predicted by an anomalous Pioneer-type acceleration which could, e.g., be induced by an extra-potential linear with distance like $\delta\Phi_N$ by Jaekel and Reynaud (2006). We not only used the RAS IAA EPM2004 ephemerides, as already done in previous works, but the NASA JPL DE405, DE200 and DE202 ephemerides and the VSOP82 theory as well. This should be sufficient to rule out, among other things, the presence of the first anomalous potential $\delta\Phi_N$ of the Jaekel and Reynaud (2006) model. Their second anomalous potential $\delta\Phi_P$, which would be able to reproduce the behavior of the Pioneer probes by assuming a quadratic dependence with distance, would also affect the orbital motion of the planets of the Solar System with extra-perihelion rates of tens-hundreds of arcseconds per century, at least in the regions in which the Pioneer anomaly manifested itself in its presently known form. Anomalous perihelion precessions of so large size are completely ruled out by the latest RAS IAA determinations of the perihelion rates for Jupiter, Saturn and Uranus by more than 10 sigmas, even after re-scaling by a factor 10 the formal errors released by Pitjeva.

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the possibility that the linear dependence holds at distance explored by Pioneer probes [...]"

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